AP® Calculus BC

2023 Free Response Questions

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_  **Score: \_\_\_\_ / 54**

**Calculator Active | Score: \_\_\_\_ / 9**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| (seconds) | 0 | 60 | 90 | 120 | 135 | 150 |
| (gallons per second) | 0 | 0.1 | 0.15 | 0.1 | 0.05 | 0 |

1. A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function , where is measured in gallons per second and is measured in seconds since pumping began. Selected values of are given in the table.
2. Using correct units, interpret the meaning of in the context of the problem. Use a right Riemann sum with the three subintervals [60, 90], [90, 120], and [120, 135] to approximate the value of .
3. Must there exist a value of , for , such that ? Justify your answer.
4. The rate of flow of gasoline, in gallons per second, can also be modeled by for . Using this model, find the average rate of flow of gasoline over the time interval . Show the setup for your calculations.
5. Using the model defined in part (c), find the value of . Interpret the meaning of your answer in the context of the problem.

**Calculator Active | Score: \_\_\_\_ / 9**

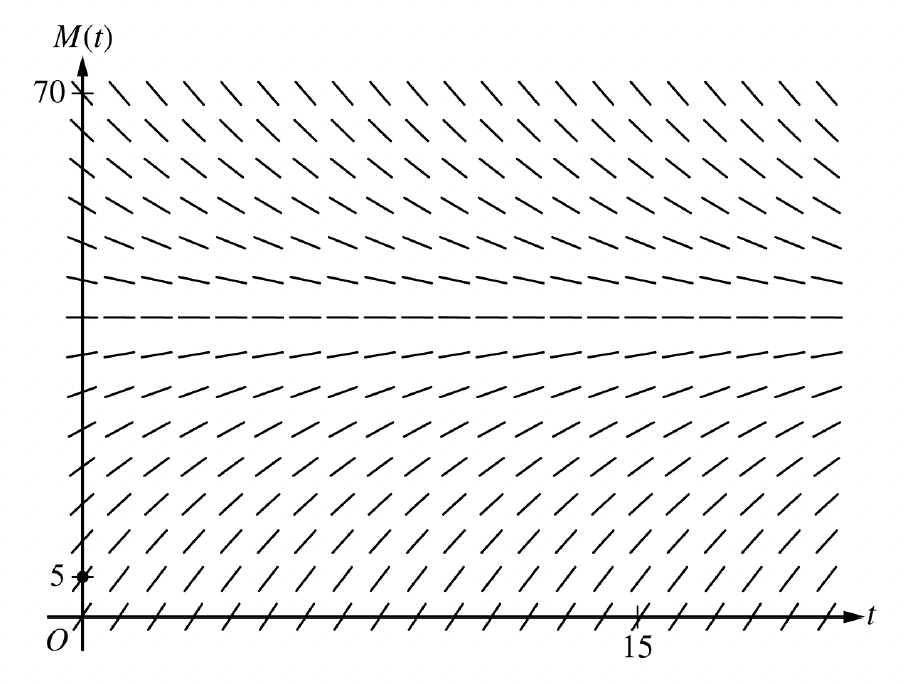
A line graph with numbers and a line

Description automatically generated

1. For , a particle is moving along the curve shown so that its position at time is , where is not explicitly given and . It is know that . At time , the particle is at position .
2. Find the acceleration vector of the particle at time . Show the setup for your calculations.
3. For , find the first time at which the speed of the particle is 1.5. Show the work, that leads to your answer.
4. Find the slope of the line tangent to the path of the particle at time . Find the -coordinate of the position of the particle at time . Show the work that leads to your answers.
5. Find the total distance traveled by the particle over the time interval . Show the setup for your calculations.

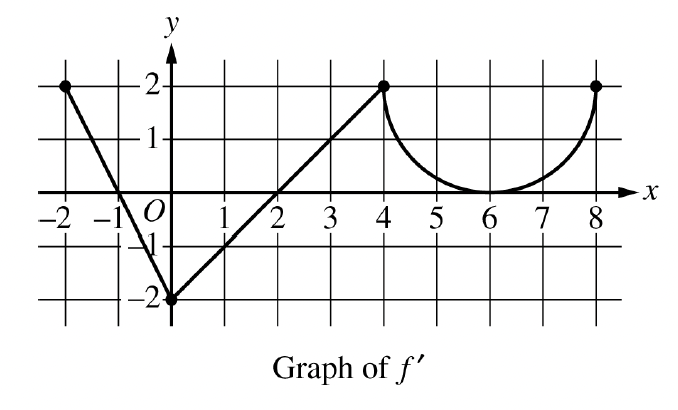
**Non-Calculator | Score: \_\_\_\_ / 9**

1. A bottle of milk is taken out of the refrigerator and placed in a pan of hot water to be warmed. The increasing function models the temperature of the milk at time , where is measured in degrees Celsius (°C) and is the number of minutes since the bottle was placed in the pan. satisfies the differential equation . At time , the temperature of the milk is 5°C. It can be shown that for all values of .
2. A slope field for the differential equation is shown. Sketch the solution curve through the point (0, 5).



1. Use the line tangent to the graph of at to approximate , the temperature of the milk at the time minutes.
2. Write an expression for in terms of . Use to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of . Give a reason for your answer.
3. Use separation of variables to find an expression for , the particular solution to the differential equation with initial condition .

**Non-Calculator | Score: \_\_\_\_ / 9**



1. The function is defined on the closed interval and satisfies . The graph of , the derivative of , consists of two line segments and a semicircle, as shown in the figure.
2. Does have a relative minimum, a relative maximum, or neither at ? Give a reason for your answer.
3. On what open intervals, if any, is the graph of concave down? Give a reason for your answer.
4. Find the value of , or show that it doesn’t exist. Justify your answer.
5. Find the absolute minimum value of on the closed interval . Justify your answer.

**Non-Calculator | Score: \_\_\_\_ / 9**

A graph of a function

Description automatically generated

1. The graphs of the functions and are shown in the figure for . It is know that for . The twice-differentiable function , which is not explicitly given, satisfies and .
2. Find the area of the shaded region enclosed by the graphs of and .
3. Evaluate the improper integral , or show that that integral diverges.
4. Let be the function defined by . Find the value of .

**Non-Calculator | Score: \_\_\_\_ / 9**

1. The function has derivatives of all orders for all real numbers. It is known that , , , and .
2. Find , the fourth derivative of with respect to . Write the fourth-degree Taylor Polynomial for about . Show the work that leads to your anwer.
3. The fourth-degree Taylor polynomial for about is used to approximate . Given that for , use the Lagrange error bound to show that this approximation is within of the exact value of .
4. Let be the function such that and . Write the second-degree Taylor polynomial for about .